Attitude Stability of Dissipative Dual-Spin Spacecraft

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Theme

THE spin-axis attitude stability in torque-free environment of dual-spin spacecraft with energy dissipation in both bodies is studied by analytical methods. A new mathematical model, which assumes the high-speed rotor to be symmetrical, is introduced and analyzed. The linearized variational periodic system of equations for the model is transformed to a kinematically similar autonomous system (according to Floquet and Liapunov) which can be analyzed routinely using established procedure.

Contents

This paper presents an extension and generalization of the analytical work of Mingori[1] in developing a mathematical model for dissipative dual-spin spacecrafts with energy dissipation on both the high-speed rotor and the despun main body.

Figure 1 shows the model introduced by Mingori in which a ball-in-tube damper is used to model the energy dissipation. This damper is a practical realization of an ideal linear damped harmonic oscillator and consists of a spring-restrained point mass in a short straight viscous fluid filled tube. Mingori linearized the equations of motion of this system near an equilibrium point and obtained a periodic system of equations. He made a complete stability analysis using the Routh criterion for two special cases: energy dissipation either only on the rotor, or only on the despun portion, in both of which the periodic coefficients of the original system vanish. Mingori, and several other researchers following him, failed to present a complete analytical solution of the stability of the periodic system of equations, and the only results available are numerical computer solutions.

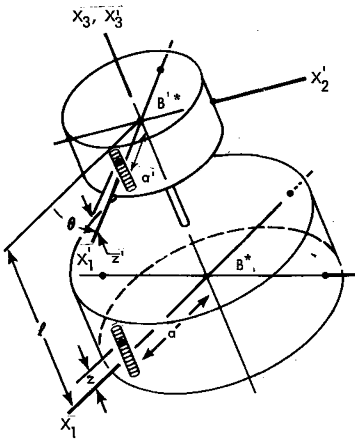
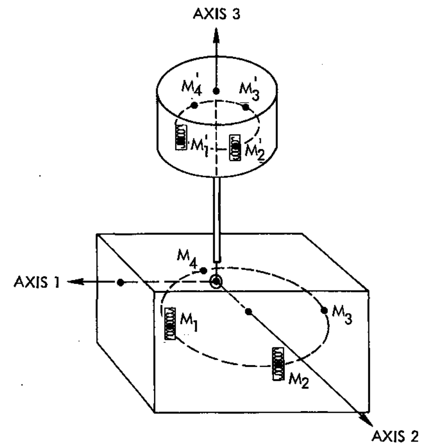


Fig. 1 Dual-spin system—Mingori's model.



BASIC DUAL SPIN CONFIGURATION

Fig. 2 Dual-spin system—proposed modification of Mingori's model.

According to the Floquet theory and the Liapunov Reducibility Theorem[2] for periodic systems, each periodic system of equations is reducible to a kinematically similar autonomous system with similar stability behavior by use of a bounded nonsingular transformation with periodic coefficients. The main difficulty in applying this result is the determination of the appropriate transformation matrix: one almost needs to know the fundamental matrix of solutions of the periodic system. However, in some cases, physical insight may suggest an obvious choice of the transformation.

All dual-spin spacecrafts known to this writer have symmetrical rotors. The energy dissipation in such rotors must be symmetrical in the two transverse axes. Mingori's model has asymmetrical rotor dissipation since he uses only one damper on the rotor. For the mathematical model to be more faithful to practical reality, one may consider a model as shown in Fig. 2. Precise mathematical specifications of the model is given in Table 1. The two rotor dampers are identical and $M_1' = M_2' = M_3' = M_4' = m'$. For convenience, it is assumed that the two dampers on the despun main body are also identical, and $M_1 = M_2 = M_3 = M_4 = m$.

The symmetry of the situation suggests that a simple rotation transformation may be used to reduce the periodic system to an autonomous one. From Table 1, θ is the angle of rotation about the common spin axis and defines the orientation of the rotor with respect to the despun main body and the displacements of the rotor damper masses are z_1' and z_2'. Two new variables, y_1 and y_2, may be defined by

$$\begin{bmatrix} z_1' \\ z_2' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{1}$$

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Table 1 Specification of the basic dissipative dual-spin configuration

Item	Mass	Rest position			Perturbed position	
Main body	M	0,	0,	1	0,	$0,1+y$
Rotor	M'	0,	0,	$1'$	0,	$0,1'+y$
Main body damper 1	m	a,	0,	d	a,	$0,d+y+z_1$
Main body damper 2	m	0,	a,	d	0,	$a,d+y+z_2$
Main body balancing	m	$-a$,	0,	d	$-a$,	$0,d+y$
masses	m	0,	$-a$,	d	0,	$-a,d+y$
Rotor damper 1	m'	a',	0,	d'	$a'\cos\theta$,	$a'\sin\theta,d'+y+z_1'$
Rotor damper 2	m'	0,	a',	d'	$-a'\sin\theta$,	$a'\cos\theta,d'+y+z_2'$
Rotor balancing	m'	$-a'$,	0,	d'	$-a'\cos\theta$,	$-a'\sin\theta,d'+y$
masses	m'	0,	$-a'$,	d'	$a'\sin\theta$,	$-a'\cos\theta,d'+y$

This transformation simply defines a rotation operation and is extremely well known and widely used. In the present context, this transformation effectively replaces the two rotor dampers by two equivalent fictitous ones, which are, as it were, fixed in the body frame. The mutual coupling terms therefore have constant coefficients. Similar usage for problems in analytical geometry is quite prevalent. Among electromechanical systems, rotating electric machinery and other transducers have been analyzed by use of this and other related transformations for several decades. An anonymous referee has pointed out that Ref. 3 reports an earlier use of the rotation transformation to dual-spin systems.

Two limitations of the applicability of this transformation should be noted. The damper masses must be negligible compared to the total mass of the system. Fortunately, this assumption is likely to be valid in most realistic cases. A more fundamental requirement is that the two rotor dampers be identical as they were assumed to be in Fig. 2. If they are not, a straightforward computation shows that the transformed equations include terms involving higher harmonics, i.e., $\cos i\theta$ and $\sin i\theta$ for integer i greater than one. The magnitudes of the coefficients multiplying these terms get bigger as the degree of asymmetry of the rotor dampers is increased. If the rotor is nearly symmetrical, the above model and the transformed equations are still useful because results are available[2] which connect the stability properties of the system $\dot{x} = Ax$ with those of $\dot{x} = [A+B(t)]x$ for constant A and small (in norm) time-varying B. However, for the extreme case where one rotor damper vanishes altogether, as in Mingori's model, the preceding transformation cannot be applied because of unavoidable singularities. This shows that the proposed modification to Mingori's model, that of using two rotor dampers in two transverse axes, is truly fundamental in nature. The proposed model is directly useful for dual-spin systems with symmetrical rotors as well as for systems with asymmetrical rotors provided that the degree of asymmetry is "small."

References

[1] Mingori, D. L., "Effects of Energy Dissipation on the Attitude Stability of Dual Spin Satellites," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 20–27.

[2] Hahn, W., *Stability of Motion*, Springer-Verlag, Berlin, 1967, pp. 296–304, 271–278.

[3] Barba, P. M., Hooker, W. W., and Leliakov, I., "Parameter Selection for a Class of Dual Spin Satellites," *Proceedings of the Symposium on Attitude Stabilization and Control of Dual-Spin Spacecrafts*, Aerospace Corp., Aug. 1967.